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NOTES ON THE CIRCUMSTANCES

OF A

MOVING PROJECTILE

PREPARED FOR THE USE OF STUDENTS IN MILITARY SCIENCE

BY

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In the following paper the attempt is made to exclude mathematics as much as possible.

The principles of the gyroscope as applied to a projectile are set forth by Captain Richmond, 2d Artillery, in *Journal of the Military Service Institution* for January, 1891.

Apology is made for the crudeness of the figures.

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March, [illegible].

A MOVING PROJECTILE

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1. We are taught by mechanics that a body set in motion will continue indefinitely in a straight line, unless retarded or turned aside by some external force.

A projectile leaves the muzzle of the gun moving in the prolongation of the axis, yet immediately diverges therefrom, and its centre of gravity in its path through the air describes a curve of (in general) double curvature. To this curve we apply the name **trajectory**.

To discuss the circumstances of a projectile moving along its trajectory, we must begin by considering the various forces which have caused it to depart from a rectilinear path.

2. Before the projectile clears the muzzle it is acted upon by the propelling force, and an increase in this force increases the velocity of the projectile and consequent distance to which it is thrown.

Whether at rest or in motion, all matter is acted upon by the force of gravity, and no velocity that we can now give to a projectile will prevent it from eventually falling to the earth.

The air offers resistance to all moving projectiles.

The trajectory is then the path described under the influence of the propelling force, gravity, and the resistance of the air. These are to be considered in order.

3. **The Propelling Force.** The pressure of the expanding gases in the bore in rear of the projectile imparts to it the velocity with which it emerges from the muzzle. This muzzle
initial velocity has for a long time been regarded as the high-
elocity reached by the projectile, but the recent experiments of
ore and Squier show that in the case of our field-piece (the

only gun with which the experiments were conducted) the velocity increases for from six to eight feet beyond the muzzle, the increase amounting to as much as 40 f.s.

4. The Effect of Mass. Suppose that we take a small leaden ball and a wooden one of exactly the same size, and throw them, using the same exertion. The hand describes an arc around the shoulder as a centre, and in the same time in both cases, therefore the balls leave the hand with the same initial velocity. The resistance of the air is eliminated from our comparison, as it is the same in each. The leaden ball is not like a rocket carrying within itself means of keeping up its velocity, yet it is thrown twice as far as its wooden counterpart. How do we account for this? It is because, bulk for bulk, the lead contains more matter than the wood,—its mass is greater.

The mass of a body is proportional to its weight. By some the terms mass and inertia are used interchangeably. The ancients called inertia *vis*, a force, and so Newton spoke of it. Now we are told that it is not a force at all, but a property. Whatever it may be, we can avoid the point and say that inertia is that by which all bodies resist any change in their state as regards rest or motion.

In other words, all bodies resist being put in motion, and after they are once put in motion they resist being accelerated, retarded, turned aside, or stopped, and in every case this resistance varies directly with their weight.

This explains the behavior of the lead and wooden balls. They started with the same velocity and met the same resistance from the air, but the leaden ball, having the greater mass, gave more resistance to being stopped, and hence attained a greater range.

Bearing this principle in mind, a western inventor has advanced a novel proposition. He proposes that the military when employed against mobs should be furnished with cartridges with aluminium bullets. At one hundred yards these would be just as effective as lead, but being almost one-third as light, they lose their velocity rapidly, and at two hundred yards are spent and

harmless. The killing of innocent citizens at a distance would thus be avoided.

5. Materials used for Projectiles. This also explains why heavy materials have always been used for projectiles. *Stone* balls were used by the Turks as late as 1807. *Lead* was the first metal employed, and still holds its own in small arms, but is too soft to be used in cannon, and is even partially melted by the heat of discharge. *Wrought iron* also is too soft, and is expensive to handle, since it must be forged into shape. For all purposes except armor piercing, *cast iron* is employed. It is cheap, easily worked, and when cast in a chill mould is sufficiently hard to be used against wrought iron armor. It is, however, shattered on steel or chilled iron armor. *Steel* projectiles are made both of cast steel and forged steel. The forged are the more expensive, especially when alloyed with chromium and oil-tempered, but are the best. The last lot of 12-inch armor-piercing shells purchased by our government cost $199 apiece. A 9-inch Whitworth forged steel shell costing $100, or twelve times as much as a similar cast iron one, has been fired three times through wrought iron twelve inches thick.

6. Gravity. Gravity causes an unsupported body to fall 16.1 feet in the first second of time, and accelerates its downward velocity by 32.2 feet for every successive second. We cannot dodge gravity, and no matter what velocity we give to a bullet fired from a horizontal barrel, at the end of one second the bullet will be 16.1 feet below this line. It is a popular misconception that the bullets of many rifles rise when they leave the muzzle. It is true that revolvers often "jump" and cause the bullet to fly high, but this jump occurs before the bullet has left the muzzle. The bullet from a rifle begins to move downward the very instant that it clears the muzzle, and this downward motion is utterly independent of the forward one.

Manufacturers of rifles, considering one hundred yards to be the most largely used range, have arbitrarily assumed that as point blank, and so build their rifles that when the sights are held on an object at that distance, the bullet will reach the spot aimed at.

There is but one way in which this can be done; that is, by making the notch of the rear sight farther from the axis of the bore than the tip of the front sight. In cases where the sights are of the same height, it will be found that the barrel is thicker at the breech than at the muzzle, and is in fact a frustum of an elongated cone.

This principle of construction is well shown in our recently discarded Springfield rifle, when we came to gallery practice at a range of 50 feet. Here we must aim eight inches below the bull's-eye in order to strike it.

It is hard to realize that with a Springfield rifle sighted for 400 yards, the axis of the bore is pointed about 16 feet above the object aimed at, but a simple calculation will prove it to be so. In firing at an object 1,000 yards distant, the bullet actually rises over 43 feet.

It is seen that by elevating the axis of the bore we cause the projectile to rise higher into the air, consequently a greater time elapses before it falls to the earth, and as during this time it continues to move forward, the range is correspondingly increased.

The angle made by the axis of the bore with the horizontal plane is called the angle of elevation.

In general the same range can be attained by two different angles of elevation, one being the complement of the other, and in the limiting case of 45°, where the angle and its complement are equal, the range is the maximum. This is so nearly true for the lower velocities that muzzle-loading mortars are fired at a constant angle of 45° elevation, and variations in the range produced by varying the amount of powder in the charge.

The greater the velocity that we give to our bullet, the greater the distance that it will traverse in one second, and the sixteen feet of drop being distributed over a greater horizontal distance, the trajectory will be flatter. This has an important bearing upon the "dangerous space."

This principle is such a matter of every-day observation that the velocity of a moving body is instinctively judged by the flatness of its trajectory. A base ball, "hot" from the bat, comes whist-

ling through the air in almost a straight line. Skilled pitchers take advantage of this, and by using what is known as a slow ball,—that is, by giving just sufficient up-curve to a ball of low velocity to compensate for its downward motion due to gravity,— deceive the batsman, who, seeing the ball approaching on a flat trajectory, thinks it has great speed, and strikes before it reaches him.

7. The Resistance of the Air. The resistance of the air varies with (1) the density of the air, (2) the velocity of the projectile, (3) its cross section, and (4) its form.

The density of the air varies with the barometric pressure, the temperature, and the amount of moisture in the air. These are considered when great exactness is required.

8. The Velocity. There is no simple law connecting resistance and velocity. The resistance increases with the velocity, but not in the same ratio; thus, for certain velocities the resistance increases as the square, and for higher velocities as the cube. The determination of the relation between resistance and velocity has been the object of countless experiments.

To give an idea of the resistance of the air against a projectile: "A wind of one hundred miles per hour is a hurricane that tears up trees and sweeps buildings before it. The pressure on a projectile moving at the recently attained high velocities is over eighty times as great as that assigned to the hurricane."

9. The cross section. The resistance of the air varies directly as the area of the cross section of the projectile; therefore, if we can reduce this area we can reduce the resistance.

By reducing the size of the projectile we reduce its cross section, but we also reduce its mass in a more rapid ratio, and we have seen that it important to retain the mass; hence this plan must be rejected.

Another plan remains: elongate the projectile, exposing less penetrating surface to the air, but retaining the mass by piling it on in rear of this surface. It is true that there is friction of the air upon the sides of elongated projectiles, but it is insignificant when compared to that upon the head.

To what extent shall this elongation be carried? Shall the

projectiles be rolled out until they become immensely long needles? Practical considerations establish a limit. First, the weakening of the projectile by excessive elongation; second, the need of a cavity in our projectiles for charges of explosives; third, the increasing difficulty of firing such projectiles end on.

The length of a projectile is generally expressed in terms of its diameter or "calibre," and about three and a half calibres is the maximum length for heavy projectiles. A twelve-inch projectile would thus be 42 inches long.

10. Methods of firing elongated projectiles end on. The first method is by taking advantage of the fact that the force of propulsion acts through the centre of gravity of the projectile, and the resistance of the air through the centre of figure. This is well shown in the case of a rocket. Without a stick a rocket will wander aimlessly in any and all directions; but with its stick, as soon as it begins to turn out of its path, the stick is brought cross-wise to the air, and the resistance drives it back, just as the wind causes a weathercock to face towards it.

Such also is the principle of the arrow of the savage. Whilst the shaft of the arrow adds some weight to the whole and thus increases its mass, still, if he could fire the head alone from his bow string, he would gladly dispense with the shaft. But the shaft is essential to keep the arrow-head point on.

We have gone beyond the savage. The bullet is our arrow-head and we have learned to fire it without a shaft.

This first method is but rarely used. There is another and a better one.

11. Rotation. A body set to rotate about an axis resists by its inertia not only any attempt to stop the rotation, but any attempt to change the angle of the plane of rotation, and this resistance increases with the velocity of rotation.

This principle is best exhibited by the gyroscope. The figure (1) represents an early form consisting of a metal wheel turning freely in the arms of a semi-circular support which in turn moves freely around a pivot at **p**. The wheel now being caused to spin rapidly, the square base can be taken in the hand and turned to

the right or the left, but the wheel, not conforming to these movements, causes its support to turn about the vertical pivot and persists in rotating in the original plane.

If, therefore, we can give our elongated projectiles a sufficient velocity of rotation around their longer axis, they will continue in their course without tumbling (or turning end for end), and without deviating from the original direction.

12. Rotation, how communicated. This rotation has been imparted in several ways.

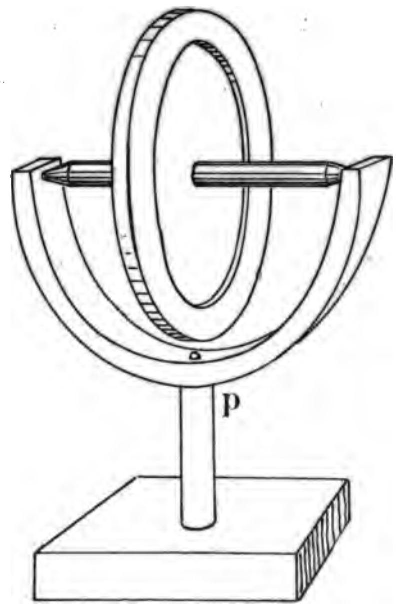

Fig. 1.

First, projectiles fired from smooth-bore guns were furnished with spiral vanes or were cast with spiral grooves. The air caused them to revolve as they moved forward. In this way the projectile of the dynamite gun is made to rotate.

Second, spiral grooves are cut in the bore of the piece and the projectile given a shape to fit these, or pieces of softer metal are attached to the projectile and take hold of the grooves in various ways.

There is a third method, not yet beyond the experimental stage. Smooth-bore guns are used and the gun itself is rotated just before firing. In a similar manner Turpin's war rockets were spun at high speed before being fired, thus becoming gyroscopic projectiles. War rockets were abandoned in 1870. One fired from Metz returned and fell in the city.

13. This motion of rotation given to the projectile causes some changes in the trajectory which it will be well to notice. A further examination of the gyroscope will enable us to explain these changes.

14. **The Gyroscope.** We will now take a different form of gyroscope, a metallic wheel and axle as before, but in this case the axle crossing a circular ring. On the ring at one extremity of the axle is a projection with a little cavity fitting loosely over an upright pivot **p**. (Fig. 2.) For simplicity's sake, in the following discussion, supposing the observer to be at **p** and looking towards the wheel, its rotation will be in the direction of the hands of a watch,—that is, from left to right.

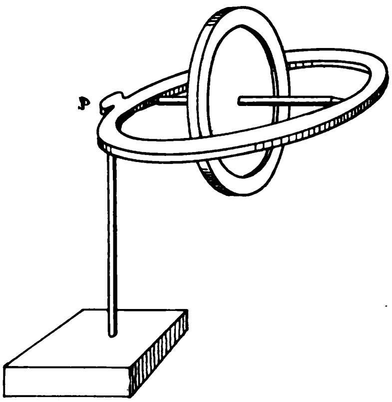

FIG. 2.

Now if the ring be held in the hand and the wheel spun rapidly and then the cavity placed over the pivot **p**, in apparent defiance of gravity the wheel will remain suspended in the air. Furthermore, it will take up a slow motion of rotation about **p**, moving with intermittent velocity to the left of the observer at **p**, and whilst doing so will fall and rise in a sinuous path, never rising higher than the original point of starting, but falling lower and lower as the wheel runs down.

15. Before attempting to explain this, let us examine a third form of gyroscope. This form differs from the preceding by the addition of an arm to the left of the pivot, upon which arm there slides an adjustable weight **W**. (Fig. 3.)

With this weight removed, the gyroscope acts just as described.

With the weight at **W** just balancing the wheel, there is no horizontal rotation.

With the weight at **w**′ overbalancing the wheel, the horizontal rotation takes place, but to the right of the observer at **p**.

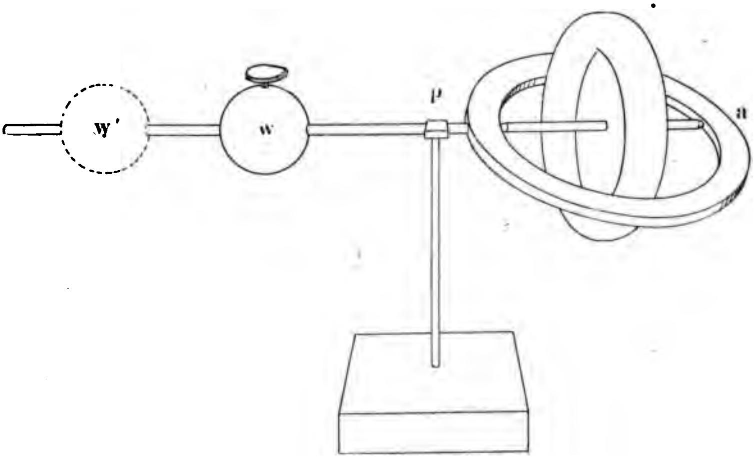

FIG. 3.

We now get our clew at once. The suspension and horizontal motion of the wheel are due (1) to the rotation of the wheel and (2) to the actual movement of the wheel like a pendulum up or down about **p**.

16. The Gyroscope explained. The figure represents the essential parts of our gyroscope. Consider the front half of the wheel. Every particle in this half has a certain downward component. Let us take any particle, as **n**. (Fig. 4.) It is moving downward in the direction **ns**. Suppose that just at this instant the wheel

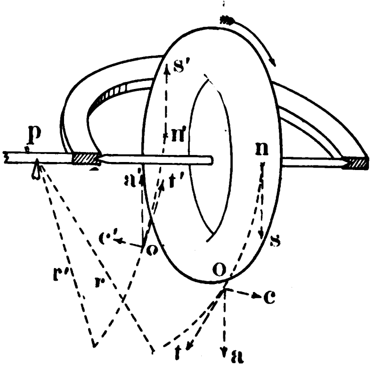

FIG. 4.

is placed upon the pivot at **p** and released. Not being supported, it begins to swing down about **p** as a centre, and **n** reaches some

other point, **o**. The wheel being rigid, the particle which at **n** was moving in the direction **ns** is now constrained to move in the direction **ot**. Its inertia causes it to resist this change in the direction of its motion, and this resistance acts in the direction **oc**.

A similar course of reasoning for any particle, **n′**, in the back half of the wheel shows that its resistance acts in the direction **o′c′**.

But every particle in the front half of the wheel acts as the particle at **n,** and every particle in the back half as the particle at **n′**.

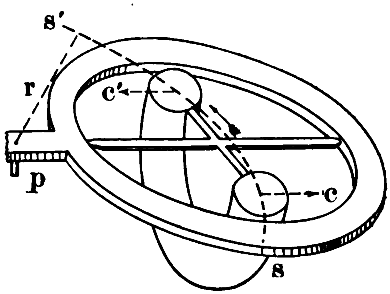

FIG. 5.

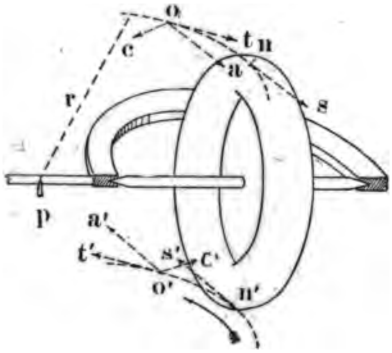

FIG. 6.

Therefore (Fig. 5.), for the front half of the wheel there is a resultant component acting in the direction **c**, and for the rear half a similar component acting in the direction **c′**. The result of this couple is to cause the system to rotate about the point **p** in the direction **ss′**.

Now suppose the wheel to consist of an upper and a lower half. Every particle in the upper half has a certain component to the front.

Let **n** be any particle moving in the direction **ns**. (Fig. 6.) As the wheel, in its horizontal rotation moves back **n** reaches some other point, **o**, and is constrained to move in the direction **ot**. Resistance is developed acting in the direction **oc**.

Similarly, for the lower half of the wheel resistance is developed in the direction **o′ c′**.

Continuing the previous course of reasoning, it is seen that a couple is developed which tends to lift the wheel about the point **p**. (Fig. 7.)

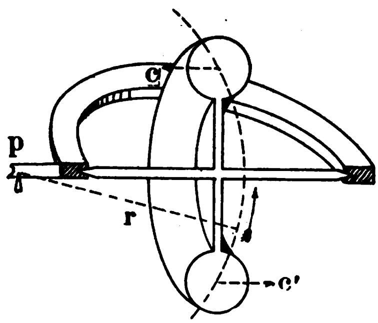

Fig. 7.

The sinuous motion and intermittent horizontal velocity of the wheel are now explained. When released it first drops. This causes it to move backward. This backward movement develops the lifting couple and the wheel rises, apparently pauses, drops again and resumes its motion to the rear, and so on.

In the case where the wheel is just balanced, it does not drop when released, no couples are developed and there is no horizontal motion.

In the case where the wheel is overbalanced by the weight, when it is released it does not drop, but rises; hence couples are developed in an opposite direction, and horizontal motion takes place to the right.

Had the rotation of the wheel been from right to left, these phenomena would have been reversed. The right-handed rotation was taken because that is the customary rotation given to projectiles.

17. Dismissing this explanation now, the important points to remember are these (See Fig. 3): If the point **a** falls, the wheel moves to the left; if it rises, the wheel moves to the right.

18. Let us now see what bearing this has upon a rotating projectile.

Upon leaving the muzzle, the axis of the projectile is tangent to the trajectory, but as the trajectory curves, the axis, because of the rotation, remains parallel to its original direction. (Fig. 8.) Thus the projectile, viewed from various points along the tra-

jectory, exposes more and more of its side to the front (Fig. 9), and at long ranges may even strike upon its side.

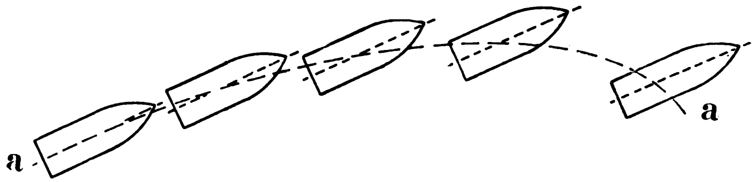

FIG. 8.

The propelling force acts through the centre of gravity, the resistance of the air acts in the direction of the tangent to the trajectory and through the centre of figure. Viewed from the axis of the projectile these two centres coincide, but from an oblique position the centre of figure is in general in front of the centre of gravity. (In a vertical section of a 3.2-inch shot it is .45 of an inch above the centre of gravity.) From Fig. 10 it is evident that the result is to lift the point of the projectile upward in the direction of **s**, about **c** as a centre. But, from par. 17, this causes the point of the projectile to move off to the right. The projectile, now thrown obliquely to the direction of its motion, sheers off bodily to the right.

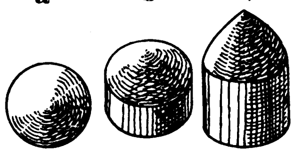

FIG. 9.

19. Drift. This divergence from the plane of departure is called **drift.** It is evident that the causes which lead the point of the projectile to move to the right in the first place will next lower it and finally move it back to the left, but usually the end of the trajectory is reached before this result occurs.

It is also evident that in those few projectiles whose centre of figure is in rear of the centre of gravity, the drift will be to the left; so also in guns whose rifling is left-handed.

20. A consideration of the foregoing will probably suggest a paradox: thus, first, we give a projectile a motion of rotation to keep it point on and increase its accuracy; and second, because

of the rotation the projectile does not strike point on, and diverges to one side.

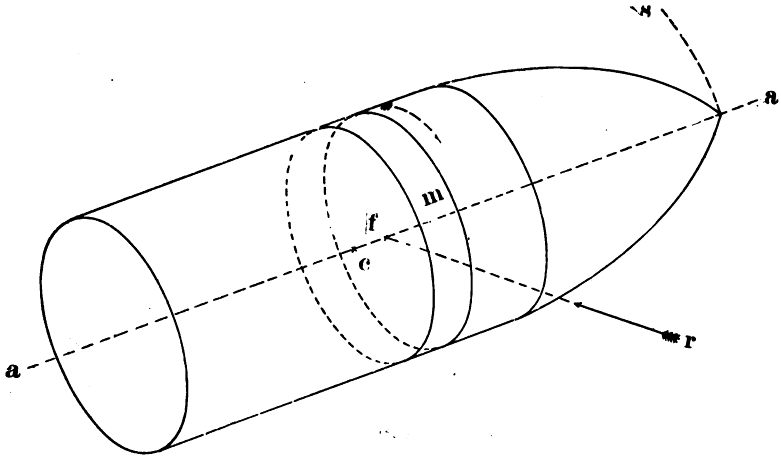

FIG. 10.

It will be sufficient to remark that whilst in the case of a smooth bore there was no telling to which side or how much a projectile would diverge, in a rifle the divergence is constant, and can be automatically allowed for by the sights, as is the case in the sight of the Springfield rifle.

21. Rotation of spherical projectiles. Spherical projectiles are not usually fired from a rifle. However, in a smooth bore the shot cannot fit air-tight, but a certain amount of space, "windage," is allowed; therefore, in moving along the barrel these shot roll over and over and bound from side to side (balloting), and leave the muzzle spinning, the direction of rotation depending upon whether the last point struck was the top, bottom or side of the bore. The result is that they travel on uncertain curved paths, the cause being the same as that which curves a base ball. In general, a ball moving through the air and spinning at the same time, has a portion of its surface moving against the air through which it passes and another portion moving with it. There is

therefore more friction on one side than on the other, and the ball moves off to the side upon which the drag acts. The exceptional case is where the axis of rotation corresponds with the axis of the bore. Hence the rifled muzzle-loading small-arms, although firing a spherical ball, were much more accurate than smooth-bores.

In spherical solid shot the centres of gravity and figure coincide, but in spherical shell, unless the cavity be concentric with the exterior, the metal will be thicker on one side than another, these centres will not coincide, and the shell when fired will wobble. It is therefore an important point in inspecting such shell to determine whether the cavity is exactly in the centre or not.

22. **Form of projectile.** The resistance of the air varies with the form of the projectile.

What is the best form for the head of a projectile,—that is, the form that will pierce the air with the greatest ease?

Several forms have been deduced theoretically, but these theoretical forms have not proven to be the best.

Attempts have been made to find the best form, as follows. Cylinders of ice towed through the water wear away to a cigar-shape, a shape of supposed least resistance.

We now rely upon the results of the experiments conducted by Dr. Bashforth under the auspices of the British Government, from 1865 to 1880. He used projectiles with heads of five different shapes, viz., flat, spherical, hemispheroidal, ogival $r = 1$, and ogival $r = 2$. The hemispheroidal and ogival $r = 2$ gave the least resistance.

It is found that the actual point of the projectile has but slight effect upon the resistance, but that the shape of the curved portion joining the cylindrical body has great effect.

It has been shown by Krupp and others that the shape of the rear end of the projectile has also an important influence upon the resistance. If gently curved, the eddy or vacuum in rear is avoided to some extent. In this connection interesting studies have been made, by means of electricity and photography combined, of the flow of air along the sides and rear of a projectile.

However, practical considerations forbid the general use of projectiles of such shape, and they are used only in the Whitworth gun and in some of the smaller Hotchkiss rapid-fire guns.

Peculiar forms have been given to the heads of projectiles to enable them to catch better hold upon armor, to prevent ricochet, etc.

23. In addition to the foregoing, the trajectory is affected by the wind.

24. It is also affected by the rotation of the earth. This causes all projectiles to deviate to the right.